Equilibrium orientation of an ellipsoidal particle inside a dielectric medium with a finite electric conductivity in the external electric field

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We study the stability of the orientation of an ellipsoidal dielectric particle immersed into a host dielectric medium under the action of the external electric field. It is assumed that the particle and the host medium have a finite electric conductivity. We demonstrate that an equilibrium orientation of the ellipsoidal particle changes with time in a stationary electric field with a constant direction. It was found that during time interval T_1 an equilibrium orientation of the spheroidal particle with a finite electric conductivity remains the same as the equilibrium orientation of an ideal dielectric particle. During time interval T_2, where $T=T_1+T_2$ is a period of the external electric field, the equilibrium orientation of the axis of symmetry of the particle is normal to the initial equilibrium direction.

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I. INTRODUCTION

The dynamics of solid or liquid particles in a host medium under the action of an external electric field is of theoretical and technological interest. Technological application includes manipulation of microparticles in biotechnology and genetic engineering [1], nanotechnology [2,3], and noncontact measurements of physical properties of particles. Interaction of an external electric field with an inclusion embedded into a host medium is important for understanding the mechanisms of the electric breakdown of dielectrics, in atmospheric physics and aerosol dynamics [4–6]. The results obtained in numerous theoretical and experimental studies on particle dynamics under the action of the external electric field were summarized in several survey papers and monographs [6–9].

One of the issues that warrant theoretical and experimental studies is the rotation of liquid or solid particles embedded into a weakly conducting host medium. This issue has been considered in a number of publications for the case when a particle has a spherical and a spheroidal shape (see, e.g., [10–14]), and the rotation of ellipsoidal particles was analyzed in [15,16]. These studies were concerned mainly with applications, and some important aspects of the dynamics of particles in the external electric field were not addressed. Rotation of ellipsoidal particles with a shell in the nonstationary external field was studied in [15,16] using a simplified approach that did not require a comprehensive analysis of the dynamics of the particle. In this study we obtained a general expression for an instantaneous moment of forces acting at an ellipsoidal particles as a function of the orientation of its principal axes.

For a case of an ideal dielectric the mathematical formulation of the problem is known [17,18]. The situation is different for the case of a particle with a finite electric conductivity. Indeed, here an instantaneous moment of forces acting at a particle depends not only upon its instantaneous orientation but also on its orientation during the earlier time moments. The reason for this behavior is as follows. The total torque $\vec{M}$ acting upon a dielectric ellipsoidal particle with a finite conductivity is the sum of two terms, $\vec{M}=\vec{M}_\varepsilon+\vec{M}_\sigma$. The first term is $\vec{M}_\varepsilon=\vec{P}_\varepsilon\times\vec{E}_0$, where $\vec{P}_\varepsilon$ is a dipole moment determined by the initial polarization of the medium and $\vec{E}_0$ is an applied electric field. The second term is $\vec{M}_\sigma=\vec{P}_\sigma\times\vec{E}_0$, where $\vec{P}_\sigma$ is a dipole moment caused by a flow of an electric charge from the external source to the surface of a particle. The dipole moment $\vec{P}_\varepsilon$ and the torque $\vec{M}_\varepsilon$ settle during a short time interval of a local relaxation while the dipole moment $\vec{P}_\sigma$ and the torque $\vec{M}_\sigma$ settle during time τ_σ of a macroscopic relaxation that depends upon the conductivities of particle and a medium. If an applied electric field is normal to the axis of symmetry, for the case of an ideal dielectric the total dipole moment $\vec{P}=\vec{P}_\varepsilon$ and $\vec{M}=\vec{M}_\varepsilon=0$. For the case of a nonideal dielectric, the dipole moment associated with a free charge $\vec{P}_\sigma$ of a rotating particle is not aligned with the applied electric field because of the finite relaxation time so that $\vec{M}_\sigma=\vec{P}_\sigma\times\vec{E}_0\neq\vec{0}$. This difference in the directions of the external electric field and the dipole moment $\vec{P}_\sigma$ is associated with Quincke rotation that was extensively discussed in the literature [7,10,13,14]. In this study we investigate a torque acting at a stationary particle as a function of its orientation (direction of its axis of symmetry) with respect to the applied electric field. Thus we assumed that the angular velocity of a particle $\vec{\Omega}=\vec{0}$.

We show that in a medium with a finite electric conductivity, a torque acting at the particle in a stationary electric field can change the orientation of a particle even when the direction of the field is fixed. Thus, if initially the particle was in a state of a stable equilibrium, then after some time the initial orientation of the particle loses its stability. Our analysis shows that there exist two time intervals, T_1 and T_2,

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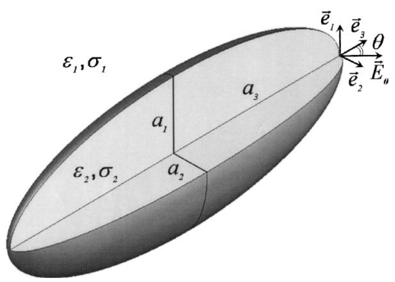

FIG. 1. Ellipsoid with semiaxes a_1, a_2, a_3 ($a_1 = a_2 < a_3$: prolate spheroid; $a_1 = a_2 > a_3$: oblate spheroid) and electric permittivity ε_2 and conductivity σ_2 inside a host medium with permittivity ε_1 in the external electric field $\vec{E}_0$.

such that during time T_1 the stable orientation of the particle is the same as for the case of an ideal dielectric. During time interval T_2, where $T = T_1 + T_2$ is a period of the external electric field, the direction of stable orientation is normal to that for the case of an ideal dielectric.

This paper is organized as follows. In Sec. II we present a mathematical formulation of the problem and discuss the underlying physics. Special attention is given to those features in the formulation of the problem that arise due to a finite conductivity of a host medium. In particular, we elucidate the physical aspects that constitute the difference between the problem for the case of a weakly conducting dielectric and an ideal dielectric case. In Sec. III we calculate the basic parameters required to determine the electric field and the electric current of a dielectric ellipsoid with permittivity ε_2 and conductivity σ_2 that is embedded into a host medium with permittivity ε_1 and conductivity σ_1. In Sec. IV we investigate stability of the orientation of particle in the external electric field.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider an ellipsoidal particle with permittivity ε_2 and conductivity σ_2 embedded into a host medium with permittivity ε_1 and conductivity σ_1 in the external electric field with a strength $\vec{E}_0$ (see Fig. 1).

In a conducting medium a potential component of an electric field $\vec{E} = -\vec{\nabla}\varphi$ is determined by the following system of equations:

$$\vec{\nabla} \cdot \vec{D} = \rho_{ex},$$

$$\frac{\partial \rho_{ex}}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0, \qquad (1)$$

where electrostatic induction $\vec{D}$ and electric current density $\vec{j}$ are determined by the following relations:

$$\vec{D} = \varepsilon_0 \varepsilon \vec{E}, \quad \vec{j} = \sigma \vec{E}, \quad \vec{E} = -\vec{\nabla}\varphi. \qquad (2)$$

Hereafter we assume that a particle is at rest. Formula (2) for an electrostatic induction implies that a characteristic time required to attain an equilibrium polarization is substantially smaller than other characteristic times in the problem. Thus the time required for a charge redistribution taking into account a finite conductivity is much larger than the characteristic time of microscopic relaxation of the dipole moments induced by local polarization.

The host medium with an embedded particle can be considered as a piecewise homogeneous medium. Since a charge is localized at the inhomogeneities, in the case of a piecewise homogeneous medium it accumulates at the interface boundaries. Density of a surface free charge γ is determined by the following relations:

$$\int \rho_{ex} dV = \int \gamma dS \text{ or } \rho_{ex} = \gamma \delta(u) |\vec{\nabla} u|, \qquad (3)$$

where $\delta(u)$ is a Dirac's delta function, $u = F(x, y, z)$ and $u = 0$ is an equation of the surface. Equations (1) and (3) yield boundary conditions at the interface boundary:

$$[\vec{N} \cdot \vec{D}] = \gamma, \quad [\vec{N} \cdot \vec{j}] = -\frac{\partial \gamma}{\partial t}. \qquad (4)$$

Here $[A] = A_+ - A_-$, A_+ and A_- are values of a function A at the external and internal surfaces, respectively, and $\vec{N}$ is the external unit normal vector.

As mentioned earlier, the main difference between a weakly conducting medium and an ideal dielectric is charge transport from the external source to the interface boundary.

Time variation of a charge constitutes the principal difference between a leaky dielectric and an ideal dielectric model [17,18]. In the following section we determined time dependencies of this charge and electric potential φ.

III. ELECTRIC FIELD AND ELECTRIC CURRENT IN A MEDIUM WITH AN ELLIPSOIDAL INCLUSION

In this section we determine an electric field of an ellipsoid immersed into a medium with a finite electric conductivity. Using the obtained results we investigate variation of the electric field during electric charge flow from the external source to the surface of the ellipsoid, variation of the electric charge γ at the surface of the ellipsoid and dependence of the electric charge relaxation time upon the geometrical parameters of the ellipsoid.

Consider an ellipsoidal inclusion with the half-lengths of the axes a_1, a_2, a_3, permittivity ε_2 and electric conductivity σ_2 that is immersed instantaneously into a host medium with permittivity ε_1 and electric conductivity σ_1 in the external electric field $\vec{E}_0$ (see Fig. 1).

The solution of an electrostatic problem is performed in a system of coordinates associated with an ellipsoid. In this system of coordinates the equation of a surface of the ellipsoid and the components of the electric field are determined by the following equations:

$$u = \sum_{i=1}^{3} x_i^2 / a_i^2 - 1, \quad \vec{E} = \sum_{i=1}^{3} E_i \vec{e}_i, \quad \vec{e}_i = \vec{\nabla} x_i. \qquad (5)$$

If before the insertion of an ellipsoidal particle, the electric field was homogeneous then electric potential φ can be written in the following form:

$$\varphi = \sum_{i=1}^{3} \varphi_i = -\sum_{i=1}^{3} E_{0i}x_i(1 + F_i(\xi,t)), \tag{6}$$

where ξ (and coordinates η,ς used below) are the ellipsoidal coordinates determined through x_1,x_2,x_3 by formulas presented in [17] (Chap. 1, Sec. 4) and ξ is chosen such that $\xi=0$ corresponds to a surface of the ellipsoid $u=0$. The potential φ in each medium is determined by the Laplace equation:

$$\nabla^2\varphi = 0. \tag{7}$$

The expression for $F_i(\xi,t)$ can be written as follows:

$$F_i(\xi,t) = F_{i1}(\xi,t)\theta(\xi) + F_{i2}\theta(-\xi), \tag{8}$$

where

$$\theta(x) = \begin{cases} 1, & x \geqslant 0, \\ 0, & x < 0. \end{cases}$$

Using formula (6) and solving Eq. (7) with continuity condition for the potential φ we arrive at the following equation:

$$F_{i1}(\xi,t) = \frac{F_{i2}(t)I_i(\xi)}{I_i(0)}, \tag{9}$$

where (for details of this solution see [17], Chap. 1, Sec. 4)

$$I_i(\xi) = \int_{\xi}^{\infty} \frac{ds}{(s + a_i^2)R(s)}, \quad R(s) = \sqrt{(s + a_1^2)(s + a_2^2)(s + a_3^2)}, \tag{10}$$

and function $F_{i2}(t)$ is determined from the boundary conditions (4). In order to determine the function $F_{i2}(t)$ it is convenient to represent a free electric charge $\gamma(\eta,\varsigma,t)$ as

$$\gamma(\eta,\varsigma,t) = \sum_{i=1}^{3} \gamma_i(\eta,\varsigma,t), \tag{11}$$

where $\gamma_i(\eta,\varsigma,t)$ is a free electric charge accumulated at the surface of an ellipsoid due to the ith component of an electric field. For the case of an ellipsoid in a homogeneous external electric field $E_{0i}(t)$, it is also convenient to represent $\gamma_i(\eta,\varsigma,t)$ as

$$\gamma_i(\eta,\varsigma,t) = \frac{\varepsilon_0\varepsilon_1}{h_1(0,\eta,\varsigma)} \frac{E_{0i}(t)x_i(0,\eta,\varsigma)}{2a_i^2}\tilde{\gamma}_i(t), \tag{12}$$

where $h_1(0,\eta,\varsigma)$ is a Lamé's coefficient along the coordinate ξ at $\xi=0$, $x_i(0,\eta,\varsigma)= x_i(\xi,\eta,\varsigma)|_{\xi=0}$, $h_1(0,\eta,\varsigma)=\sqrt{\eta\varsigma}/2R(0)$ and in a Cartesian coordinate system $h_1(0)=\frac{1}{2}\Sigma_{i=1}^{3}(x_i^2/a_i^4)^{1/2}$.

In order to derive Eq. (12) for $\gamma_i(\eta,\varsigma,t)$ one can use Eq. (6) for a potential φ and Eq. (2) that determines a relation between a potential and electric field. At the surface of the ellipsoidal particle $(\vec{N}\cdot\vec{\nabla})\varphi= (1/h_1)(\partial\varphi/\partial\xi)|_{\xi=0}$ and $\partial x_i/\partial\xi|_{\xi=0}=\frac{1}{2}[x_i(0,\eta,\varsigma)/a_i^2]$. The latter relations and Eqs. (8) and (10) imply that $(\vec{N}\cdot\vec{\nabla})\varphi \propto E_{0i}(t)x_i(0)/2a_i^2h_1(0,\eta,\varsigma)$. Coefficients $\tilde{\gamma}_i(t)$ depend only upon time.

The unit normal vector at the surface of the ellipsoid $\vec{N}$ can be represented as $\vec{N}=\Sigma_{(i)}N_i\vec{e}_i$ (see, e.g., [17]) with

$$N_i = \frac{x_i(0,\eta,\varsigma)}{2a_i^2h_1(0,\eta,\varsigma)}, \quad i = 1,2,3. \tag{13}$$

Equations (11)–(13) imply the following expression for a free electric charge accumulated at the surface of the ellipsoid due to the external electric field, $\gamma(\eta,\varsigma,t)$:

$$\gamma = \vec{\gamma}' \cdot \vec{N}, \tag{14}$$

where the formula for the components of the vector $\vec{\gamma}'=\Sigma_{(i)}\gamma_i'\vec{e}_i$ reads

$$\gamma_i' = \varepsilon_0\varepsilon_1E_{0i}(t)\tilde{\gamma}_i(t). \tag{15}$$

These components have a simple physical meaning. They are equal to the magnitudes of the electric charge at the apexes of the ellipsoid: $\vec{N}_1=\vec{e}_1$, $\vec{N}_2=\vec{e}_2$, and $\vec{N}_3=\vec{e}_3$. Hereafter we will write as it is generally agreed that $\vec{N}_1=(1,0,0)$, $\vec{N}_2=(0,1,0)$, and $\vec{N}_3=(0,0,1)$. Then $\gamma_1',\gamma_2',\gamma_3'$ are the magnitudes of the free charge at these locations.

Coefficients $\tilde{\gamma}_i(t)$ are determined from the boundary conditions (4). The first equation in the boundary conditions (5) yields

$$F_{2i}(t) = -\frac{\tilde{\gamma}_i(t)n_i + f_{i\varepsilon}}{1 + f_{i\varepsilon}}, \tag{16}$$

where $n_i=R(0)I_i(0)/2$ is a depolarization factor, $0<n_i<1$, $\Sigma_{i=1}^{3}n_i=1$, $f_{i\varepsilon}=\kappa_\varepsilon n_i$, and $\kappa_\varepsilon=\varepsilon_2/\varepsilon_1-1$. The second equation in the boundary conditions (4) implies that

$$F_{2i}(t) = \frac{\dfrac{\varepsilon_0\varepsilon_1 n_i}{\sigma_1}\left(\dot{\tilde{\gamma}} + \tilde{\gamma}_i\dfrac{\dot{E}_{0i}(t)}{E_{0i}(t)}\right) - f_{i\sigma}}{1 + f_{i\sigma}}, \tag{17}$$

where $f_{i\sigma}=\kappa_\sigma n_i$ and $\kappa_\sigma=\sigma_2/\sigma_1-1$.

Formulas (16) and (17) yield the following equation for $\tilde{\gamma}_i(t)$:

$$\dot{\tilde{\gamma}}_i(t) + \tilde{\gamma}_i(t)\left(\frac{\dot{E}_{0i}(t)}{E_{0i}(t)} + \frac{1}{\tau_i}\right) = \frac{\kappa_\sigma - \kappa_\varepsilon}{1 + f_{i\varepsilon}}\frac{1}{\tau_0}, \tag{18}$$

where $\tau_0=\varepsilon_0\varepsilon_1/\sigma_1$ and $\tau_i=\tau_0[(1+f_{i\varepsilon})/(1+f_{i\sigma})]$.

Assuming that the initial free electric charge of the particle is zero, the expression for $\tilde{\gamma}_i(t)$ can be written as

$$\tilde{\gamma}_i(t) = \tilde{\gamma}_i(\infty)\Pi_i(t), \quad \Pi_i(t) = \frac{1}{E_{0i}(t)}\frac{1}{\tau_i}\int_0^t e^{-\tau/\tau_i}E_{0i}(t-\tau)d\tau, \tag{19}$$

where $\tilde{\gamma}_i(\infty)=(\kappa_\sigma-\kappa_\varepsilon)/(1+f_{i\sigma})$. If the initial free electric charge of the particle is not zero one must account for the electric field produced by this charge. Hereafter it is assumed that initially the particle was not charged.

Formula (19) allows us to determine the functions $\Pi_i(t)$ for given $E_{0i}(t)$ and parameters τ_i that characterize a system. In a case when $E_{0i}(t)=\text{const}$ the result is presented below. In the case of a stationary field the result is given by Eq. (47). In this study we expressed the considered physical characteristics through functions $\Pi_i(t)$.

Substituting Eqs. (18) and (19) into Eq. (16) yields

$$F_{i2}(t) = \frac{1-\Pi_i(t)}{1+f_{i\varepsilon}} + \frac{\Pi_i(t)}{1+f_{i\sigma}} - 1. \tag{20}$$

The magnitudes of the electric fields and currents can be determined using formulas (2), (5), and (20):

$$\vec{E}_2 = \sum_{i=1}^{3} E_{0i}(t)\vec{e}_i\left(\frac{1-\Pi_i(t)}{1+f_{i\varepsilon}} + \frac{\Pi_i(t)}{1+f_{i\sigma}}\right),$$

$$\vec{j}_2 = \sigma_2\vec{E}_2, \quad \vec{D}_2 = \varepsilon_0\varepsilon_2\vec{E}_2. \tag{21}$$

The value of the accumulated charge at the surface can be determined from formulas (11), (12), and (19):

$$\gamma = \frac{\varepsilon_0\varepsilon_1}{h_1(0)}\sum_{i=1}^{3}\frac{E_{0i}(t)x_i(0)}{2a_i^2}\frac{\kappa_\sigma-\kappa_\varepsilon}{1+f_{i\sigma}}\Pi_i(t). \tag{22}$$

According to Eq. (21) the electric field $\vec{E}_2$ can be written as a sum of two fields, $\vec{E}_2=\vec{E}_\varepsilon+\vec{E}_\sigma$, where the field $\vec{E}_\varepsilon$ describes a renormalization of the external electric field $\vec{E}_0$ due to polarization. At the initial time, $t=0$, $\vec{E}_\varepsilon(0)$ recovers the known formula for the electric field of the dielectric ellipsoid with a permittivity ε_2 imbedded into the host medium with a permittivity ε_1 (see, e.g., Ref. [17]). The term $\vec{E}_\sigma$ describes a renormalization of the external field due to an accumulation of the electric charge at the surface. If the external field is constant, $E_{0i}=\text{const}$, then $\Pi_i(t)=1-e^{-t/\tau_i}$, and at $t\to\infty$, the configuration of the electric field is identical to the configuration of the electric field produced by an ellipsoidal inclusion with electric conductivity σ_2 imbedded into the host medium with electric conductivity σ_1.

Formula (22) describes a free charge at the surface of a particle. The total charge is

$$\gamma_c = \varepsilon_0[\vec{E}]\cdot\vec{N}. \tag{23}$$

The latter formula can be rewritten as

$$\gamma_c = \frac{\varepsilon_0}{h_1(0)}\sum_{i=1}^{3}\frac{x_i(0)E_{0i}(t)}{2a_i^2}\left(\frac{(1-\Pi_i(t))\kappa_\varepsilon}{1+f_{i\varepsilon}} + \frac{\Pi_i(t)\kappa_\sigma}{1+f_{i\sigma}}\right). \tag{24}$$

Thus at the initial time $\Pi_i(t)=0$, the total charge coincides with the polarization charge that is formed during microscopic time by local polarization of the material.

Since time variation of electric charges and currents is essentially determined by the magnitude of the relaxation times τ_i it is of interest to analyze the dependence of these relaxation times on geometrical parameters of the ellipsoid. Expression for τ_i [see formulas after Eq. (18)] yields

$$\tau_i - \tau_0 = \tau_0\frac{(\kappa_\varepsilon-\kappa_\sigma)n_i}{1+\kappa_\sigma n_i}. \tag{25}$$

Equation (25) implies that when $\kappa_\varepsilon<\kappa_\sigma$, $\tau_i<\tau_0$ for an arbitrary direction i. It is known that conditions $\tau_i>\tau_0$ or $\kappa_\varepsilon>\kappa_\sigma$ are the necessary conditions for Quincke rotation that has been extensively discussed in the literature [7,14] and is not a subject of this study. Equation (25) allows us to determine a ratio of relaxation times along different axes of the ellipsoid, a and b:

$$\frac{\tau_a-\tau_0}{\tau_b-\tau_0} = \frac{n_b^{-1}+\kappa_\sigma}{n_a^{-1}+\kappa_\sigma}. \tag{26}$$

Equations (25) and (26) and conditions $-1<\kappa_\sigma<\infty$, $0<n_i<1$ imply that when $\kappa_\varepsilon>\kappa_\sigma$ and $n_a<n_b$, then $\tau_a<\tau_b$, while when $\kappa_\varepsilon<\kappa_\sigma$ and $n_a<n_b$ then $\tau_a>\tau_b$. It is known [17] that polarization factors n_1,n_2,n_3 and half-lengths of the axes of ellipsoid a_1,a_2,a_3 are related by the following condition: when $a_1>a_2>a_3$ then $n_1<n_2<n_3$. Therefore if a relaxation time of a free electric charge inside an ellipsoid is less than a characteristic relaxation time in the host medium, $\tau_i<\tau_0$ or $\kappa_\varepsilon<\kappa_\sigma$, then relaxation of a free electric charge occurs faster along the shorter axes. Alternatively, when $\tau_i>\tau_0$, charge relaxation proceeds faster in the direction of the longer axes.

For a cylinder with the axis directed along the coordinate axis x_3, $n_1=n_2=\frac{1}{2}$ and $n_3=0$. The relaxation time along the coordinate axis x_3, $\tau_3=\tau_0$ and relaxation times along axes x_1 and x_2, $\tau_1=\tau_2=\tau_0[(\kappa_\varepsilon+2)/(\kappa_\sigma+2)]$. In the case of a thin disk with the axis directed along the coordinate axis x_3, $n_1=n_2=0$, $n_3=1$ and relaxation times are $\tau_3=\tau_0[(\kappa_\varepsilon+1)/(\kappa_\sigma+1)]$, $\tau_1=\tau_2=\tau_0$.

Polarization factors n_i can be expressed as functions of the ratios of the half-lengths of the axes of ellipsoid to a half-length of one of the axes. Hereafter we expressed n_i as $n_i=n_i(a_1',a_2')$, where $a_1'=a_1/a_3$ and $a_2'=a_2/a_3$. In Fig. 2 we showed the dependence of n_1 as a function of parameters a_1' and a_2'. Since $n_1(a_1',a_2')=n_2(a_2',a_1')$, the same set of the curves describes the dependence of $n_2(a_2',a_1')$ by a change of the parameters, $a_1'\to a_2'$ and $a_2'\to a_1'$. In Fig. 3 we showed the dependence $n_3(a_1',a_2')$ by presenting the set of curves $n_3=n_3(a_1')$ for different values of parameter a_2'.

Consider now the behavior of a total surface charge γ_c which is determined by expression (24). As in the case of a free charge expression for γ_c can be written similarly to Eq. (14):

$$\gamma_c = \vec{\gamma}_c\cdot\vec{N}, \tag{27}$$

where

$$\gamma_{ci} = \varepsilon_0 E_{oi}\left(\frac{\kappa_\varepsilon(1-\Pi_i(t))}{1+f_{i\varepsilon}} + \frac{\Pi_i(t)\kappa_\sigma}{1+f_{i\sigma}}\right). \tag{28}$$

The values γ_{ci} are the magnitudes of the total surface charge at the locations $\vec{N}_1=(1,0,0)$, $\vec{N}_2=(0,1,0)$ and $\vec{N}_3=(0,0,1)$ at the surface of the ellipsoid.

In a particular case of a sphere $n_i=\frac{1}{3}$ and the coefficients $f_{i\varepsilon}$ and $\Pi_i(t)$ are independent of the direction i. In this case $\gamma_{ci}/\gamma_{ck}=E_{0i}/E_{0k}$, and the electric field inside a sphere is directed along the external electric field. In the case of an ellipsoid $\gamma_{ci}/\gamma_{ck}\neq E_{0i}/E_{0k}$, and the direction of the internal electric field varies with time even when the direction of the external electric field $\vec{E}_0$ is constant.

Equations (27) and (28) imply that a charge at any location at the ellipsoid's surface is determined by three components γ_{ci}. In Fig. 4 we showed the time dependence of the

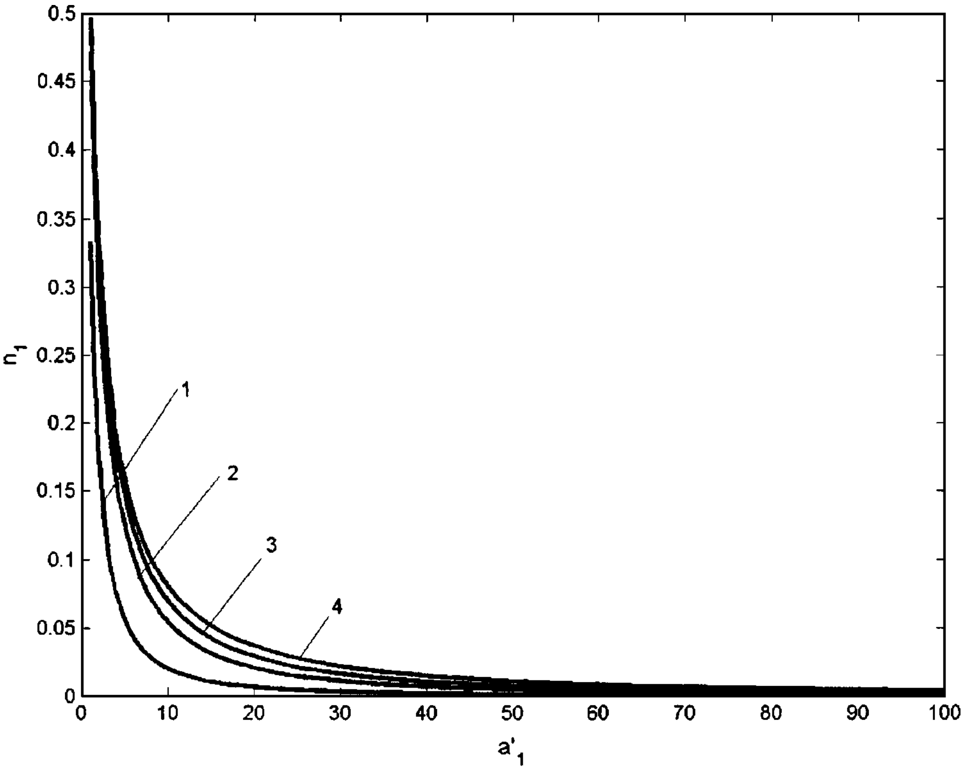

FIG. 2. Dependence of polarization factor n_1 vs the nondimensional lengths a_1' and a_2', $a_3'=1$ [(1) $a_2'=2$; (2) $a_2'=5$; (3) $a_2'=10$; (4) $a_4'=20$].

surface charge, $\gamma_{c1}(t)$, for different values of a_1' and a_2' in the case of the constant external field $\vec{E}_0$ when $\vec{E}_0 \cdot \vec{e}_2=0$ and the angle θ between $\vec{E}_0$ and $\vec{e}_3$, $\theta=\pi/4$. In Fig. 5 we showed the time dependence of the surface charge $\gamma_{c3}(t)$ for the same values of the parameters.

Inspection of Figs. 4 and 5 shows that when $\kappa_\varepsilon > \kappa_\sigma$ the total surface charge $\gamma_c(t)$ decreases with time. The cause for this behavior is that the sign of the free charge $\gamma(t)$ flowing to the surface is opposite to the sign of the polarization charge $\gamma_c(0)$. This is exactly the situation which occurs in the case of Quincke rotation. When $\kappa_\varepsilon < \kappa_\sigma$, the sign of the free charge $\gamma(t)$ flowing to the surface coincides with the sign of the polarization charge $\gamma_c(0)$, and the total surface charge $\gamma_c(t)$ grows with time.

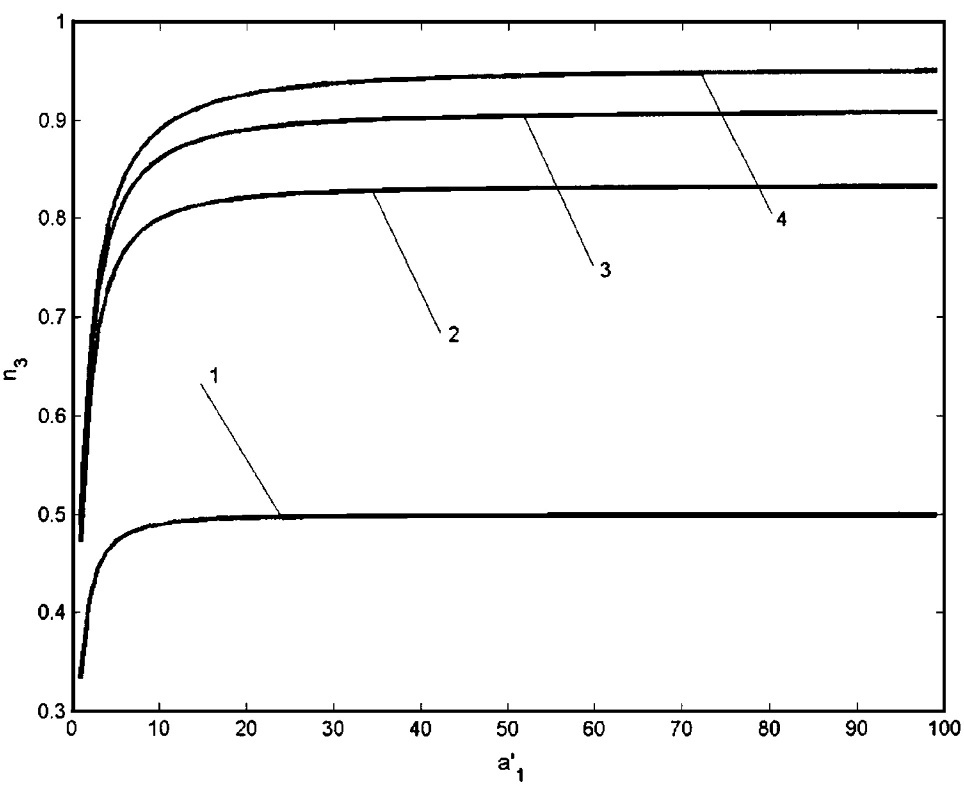

FIG. 3. Dependence of polarization factor n_3 vs the nondimensional lengths a_1' and a_2', $a_3'=1$ [(1) $a_2'=2$; (2) $a_2'=5$; (3) $a_2'=10$; (4) $a_4'=20$].

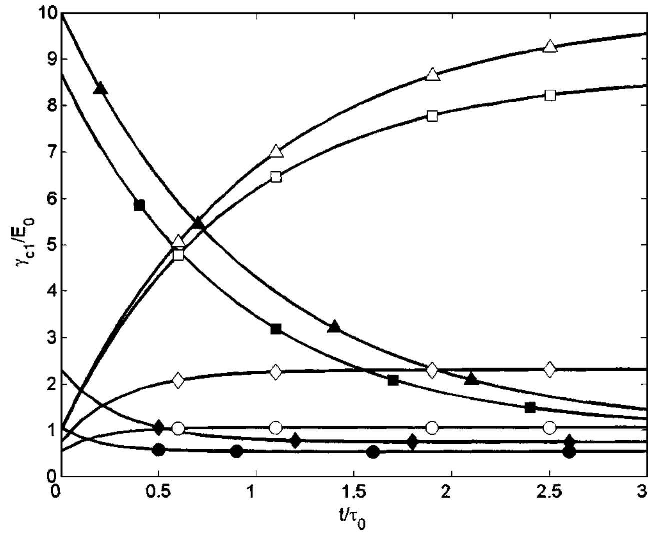

FIG. 4. Time dependence of a surface charge γ_{c1} at location $\vec{N}=(1,0,0)$ in a constant external electric field $\vec{E}_0$. Vector $\vec{E}_0$ is located in the plane $\vec{e}_1$, $\vec{e}_3$ ($\vec{E}_0\cdot\vec{e}_2=0$) and is directed by the angle $\theta=\pi/4$ with the axis of symmetry of the spheroid (○: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, prolate spheroid; □: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, oblate spheroid; ◇: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, sphere; △: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, ellipsoid; ●: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, prolate spheroid; ■: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=10$, oblate spheroid; ◆: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=10$, sphere; ▲: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, ellipsoid).

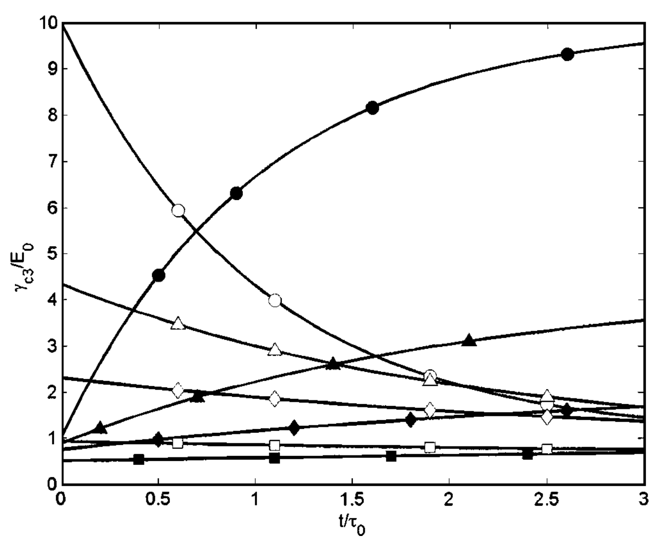

FIG. 5. Time dependence of a surface charge γ_{c3} at location $\vec{N}=(0,0,1)$ in a constant external electric field. Vector $\vec{E}_0$ is located in the plane $\vec{e}_1$, $\vec{e}_3$ ($\vec{E}_0\cdot\vec{e}_2=0$) and is directed by the angle $\theta=\pi/4$ with the axis of symmetry of the spheroid (○: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, prolate spheroid; □: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, oblate spheroid; ◇: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, sphere; △: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, ellipsoid; ●: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, prolate spheroid; ■: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=10$, oblate spheroid; ◆: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=10$, sphere; ▲: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, ellipsoid).

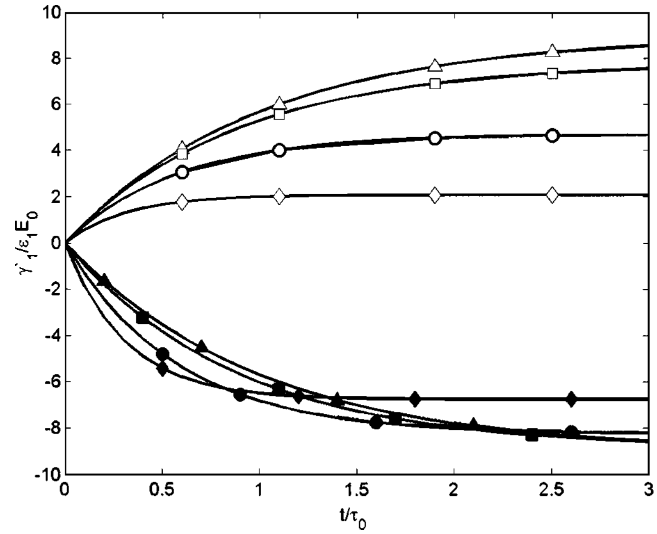

FIG. 6. Time dependence of a free charge γ_1' at location $\vec{N}=(1,0,0)$ in a constant external electric field. Vector $\vec{E}_0$ is located in the plane $\vec{e}_1$, $\vec{e}_3$ ($\vec{E}_0\cdot\vec{e}_2=0$) and is directed by the angle $\theta=\pi/4$ with the axis of symmetry of the spheroid (○: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, prolate spheroid; □: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, oblate spheroid; ◇: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, sphere; △: $-a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, ellipsoid; ●: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, prolate spheroid; ■: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=10$, oblate spheroid; ◆: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=10$, sphere; ▲: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, ellipsoid).

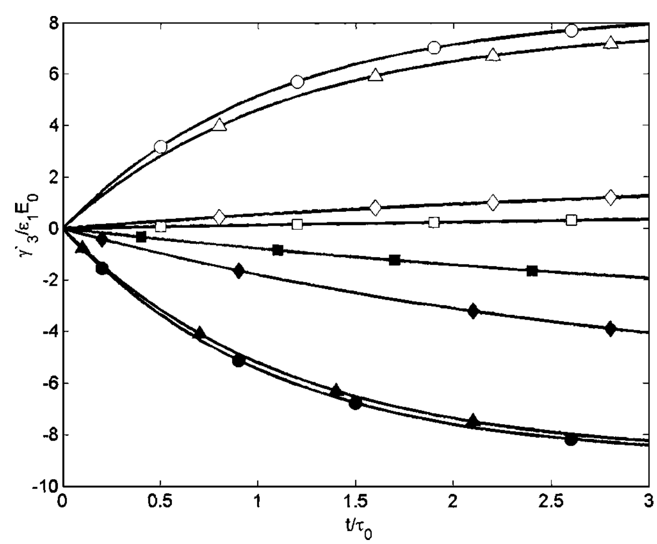

FIG. 7. Time dependence of a free charge γ_3' at location $\vec{N}=(0,0,1)$ in a constant external electric field. Vector $\vec{E}_0$ is located in the plane $\vec{e}_1$, $\vec{e}_3$ ($\vec{E}_0\cdot\vec{e}_2=0$) and is directed by the angle $\theta=\pi/4$ with the axis of symmetry of the spheroid (○: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, prolate spheroid; □: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, oblate spheroid; ◇: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, sphere; △: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, ellipsoid; ●: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, prolate spheroid; ■: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=10$, oblate spheroid; ◆: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=10$, sphere; ▲: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, ellipsoid).

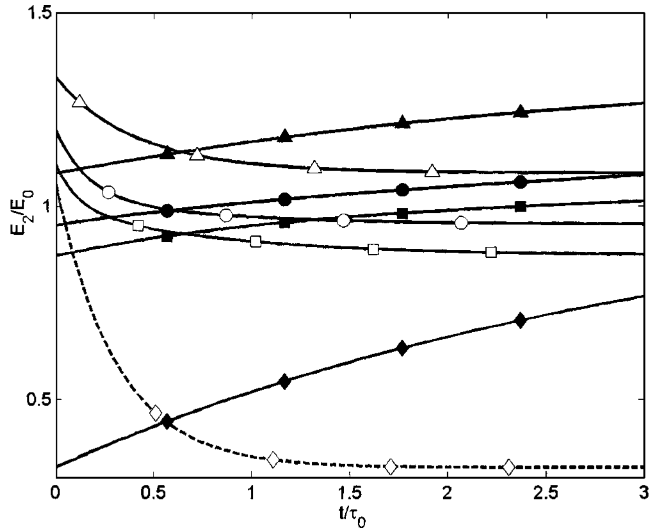

FIG. 8. Time dependence of a magnitude of the electric field inside spheroid in a constant external electric field. Vector $\vec{E}_0$ is located in the plane $\vec{e}_1$, $\vec{e}_3$ ($\vec{E}_0\cdot\vec{e}_2=0$) and is directed by the angle $\theta=\pi/4$ with the axis of symmetry of the spheroid (○: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, prolate spheroid; □; $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, oblate spheroid; ◇: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, sphere; △: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, ellipsoid; ●: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, prolate spheroid; ■: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=10$, oblate spheroid; ◆: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=10$, sphere; ▲: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, ellipsoid).

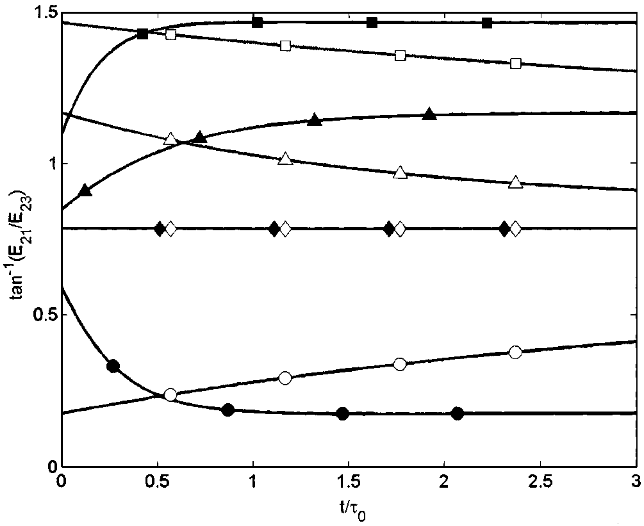

FIG. 9. Time dependence of the angle between the internal electric field and the axis of symmetry of spheroid in a constant external electric field. Vector $\vec{E}_0$ is located in the plane $\vec{e}_1$, $\vec{e}_3$ ($\vec{E}_0\cdot\vec{e}_2=0$) and is directed by the angle $\theta=\pi/4$ with the axis of symmetry of the spheroid (○: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, prolate spheroid; □: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, oblate spheroid; ◇: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, sphere; △: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=0.1$, ellipsoid; ●: $a_1'=a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, prolate spheroid; ■: $a_1'=a_2'=50$, $\kappa_\varepsilon/\kappa_\sigma=10$, oblate spheroid; ◆: $a_1'=a_2'=1$, $\kappa_\varepsilon/\kappa_\sigma=10$, sphere; ▲: $a_1'=50$, $a_2'=0.05$, $\kappa_\varepsilon/\kappa_\sigma=10$, ellipsoid).

In Figs. 6 and 7 we showed the behavior of the components γ_i' that according to Eqs. (14) and (15) is completely determined by the behavior of a surface free charge $\gamma(t)$. Inspection of these figures shows that for the same values of parameters the sign of $\gamma_i'(t)$ is opposite to the sign of $\gamma_{ci}(0)$ when $\kappa_\varepsilon>\kappa_\sigma$ and the sign of $\gamma_i'(t)$ coincides with the sign of $\gamma_{ci}(0)$ when $\kappa_\varepsilon<\kappa_\sigma$.

Time behavior of the electric field inside an ellipsoid $\vec{E}_2(t)$ is shown in Figs. 8 and 9. In Fig. 8 we showed time variation of the magnitude of the electric field $E_2(t)$ while in Fig. 9 we showed the time dependence of the angle $\alpha(t)=\tan^{-1}(E_{21}/E_{23})$. Inspection of these figures reveals that when $\kappa_\varepsilon>\kappa_\sigma$, the magnitude of the electric field $E_2(t)$ grows while for $\kappa_\varepsilon<\kappa_\sigma$ it decreases with time. The reason for this behavior is that when $\kappa_\varepsilon>\kappa_\sigma$ the electric field produced by the free charge $\gamma(t)$ is directed along the external field $\vec{E}_0$ and it partially compensates the field produced by the polarization charge.

IV. STABILITY OF THE ORIENTATION OF THE ELLIPSOIDAL PARTICLE IN THE EXTERNAL ELECTRIC FIELD

Let us now analyze the stability of the orientation of a particle by considering the dependence of the torque acting at the particle upon the orientation of the particle with respect to the direction of the external electric field. To this end we use the following formula for a torque acting at the particle:

$$\vec{M}=\varepsilon_1\vec{P}\times\vec{E}_0, \tag{29}$$

where $\vec{P}$ is a total dipole moment of the ellipsoid. Using formulas (3) the expression for $\vec{P}$ can be written as

$$\vec{P}=\int\gamma_c\vec{r}\,dS, \tag{30}$$

where integration is performed over the surface of an ellipsoid. Substituting Eq. (24) into Eq. (30) we find that

$$\vec{P}=\varepsilon_0V\sum_{i=1}^{3}E_{0i}\vec{e}_i\left(\frac{(1-\Pi_i(t))\kappa_\varepsilon}{1+f_{i\varepsilon}}+\frac{\Pi_i(t)\kappa_\sigma}{1+f_{i\sigma}}\right). \tag{31}$$

For $t\ll\tau_i$, in the constant external field $\Pi_i(t)\sim t/\tau_i$, and formula (31) recovers the known expression for a dipole moment of a dielectric ellipsoid (see [17], Chap. 2, Sec. 9). Equations (29) and (31) yield a formula for a torque acting at the ellipsoid for an arbitrary orientation of the external electric field and axes of the ellipsoid:

$$\vec{M}=\varepsilon_0\varepsilon_1V\sum_{i=1}^{3}\sum_{k=1}^{3}E_{0i}E_{0k}\varepsilon_{ikm}\vec{e}_m\left(\frac{(1-\Pi_i(t))\kappa_\varepsilon}{1+f_{i\varepsilon}}+\frac{\Pi_i(t)\kappa_\sigma}{1+f_{i\sigma}}\right), \tag{32}$$

where ε_{ikm} is a fully nonsymmetric unit tensor.

Let us consider a spheroid with a coefficient of the depolarization $n_1=n_2=\frac{1}{2}(1-n)$, $n=n_3$, $f_{1\varepsilon}=f_{2\varepsilon}$, and $f_{1\sigma}=f_{2\sigma}$. In a case of a prolate in the direction of $\vec{e}_3$ spheroid, $n_3<n_1,n_2$,

while for an oblate ellipsoid $n_3 > n_1, n_2$. The limiting cases of a cylinder ($n_3 \ll n_1, n_2$) and of a disk ($n_3 \gg n_1, n_2$) were considered earlier.

Let us define angle θ in the plane spanned by vectors $\vec{E}_0, \vec{e}_3$ (see Fig. 1). The electric field $\vec{E}_0$ can be represented as follows:

$$\vec{E}_0 = E_0 \cos\theta \vec{e}_3 - E_0 \sin\theta \vec{e}_1. \tag{33}$$

In the adopted coordinate system (see Fig. 1) a total torque acting at the particle is directed along the $\vec{e}_2$ axis, i.e., $\vec{M} = M\vec{e}_2$. Using Eqs. (32) and (33) we arrive at the following formula for M:

$$M = -\frac{\varepsilon_0\varepsilon_1 V E_0^2 \sin(2\theta)}{2}\left[\kappa_\varepsilon\left(\frac{1-\Pi_3(t)}{1+f_{3\varepsilon}} - \frac{1-\Pi_1(t)}{1+f_{1\varepsilon}}\right) + \kappa_\sigma\left(\frac{\Pi_3}{1+f_{3\sigma}} - \frac{\Pi_1}{1+f_{1\sigma}}\right)\right]. \tag{34}$$

In a constant electric field at $t=0$, $\Pi_3(0)=\Pi_1(0)$ and

$$M(0) = \frac{\varepsilon_0\varepsilon_1 V E_0^2 \sin(2\theta)}{4}\frac{\kappa_\varepsilon^2(3n-1)}{(1+f_{1\varepsilon})(1+f_{3\varepsilon})}. \tag{35}$$

Equation (35) recovers the known formula for a torque acting at the dielectric spheroid as a function of the angle between the axis of symmetry of the spheroid and the direction of the external electric field $\vec{E}_0$ (see, e.g. [17,18]). Two orientations when the torque vanishes, $\theta=0$ and $\theta=\pi/2$, correspond to stable and unstable equilibrium orientations for $n < 1/3$ and, inversely, to unstable and stable equilibrium orientations for $n > 1/3$ (for details see the Appendix).

Let us consider now stability of equilibrium orientations for $t \to \infty$. For $t/\tau_1 \gg 1$ and $t/\tau_2 \gg 1$, $\Pi_1(t)=\Pi_2(t)=1$ and $M(t\to\infty)=M_\infty$ is determined by the following formula:

$$M_\infty = \frac{\varepsilon_0\varepsilon_1 V E_0^2 \sin(2\theta)}{4}\frac{\kappa_\sigma^2(3n-1)}{(1+f_{1\sigma})(1+f_{3\sigma})}. \tag{36}$$

Comparing Eqs. (35) and (36) shows that equilibrium orientations at $t=0$ and $t\to\infty$ coincide. It can be shown that at the intermediate times $0 < t < \infty$ the sign of M_∞ is the same as the sign of $M(0)$ even in the cases with a strong anisotropy, $\tau_1 \gg \tau_3$ or $\tau_3 \gg \tau_1$.

Consider now a stationary external electric field $E_0(t) = \overline{E_0}\cos(\omega t)$. Substituting this expression into Eq. (19) yields a formula for $\Pi_i(t)$ that in the limit $t \gg \tau_1, \tau_2$ reads

$$\Pi_i(t) = \frac{\cos(\omega t) + \omega\tau_i \sin(\omega t)}{1+\omega^2\tau_i^2}\frac{1}{\cos(\omega t)}. \tag{37}$$

Substituting Eq. (37) into Eq. (34) we arrive at the following formula for the total torque acting at a particle:

$$M = M_\varepsilon + M_\sigma, \tag{38}$$

where

$$M_\varepsilon = M_0\frac{3n-1}{2}\frac{\kappa_\varepsilon^2\cos^2(\omega t)}{(1+f_{3\varepsilon})(1+f_{1\varepsilon})}, \quad M_0 = \frac{\varepsilon_0\varepsilon_1 V\overline{E_0^2}\sin(2\theta)}{2}. \tag{39}$$

The expression for M_σ can be written as

$$M_\sigma = -\frac{M_0(\kappa_\varepsilon - \kappa_\sigma)(3n-1)}{2L}[(a_1 + a_2\omega^2\tau_0^2)\cos^2(\omega t) + b_0\omega\tau_0(\kappa_\sigma b_1 + b_2\kappa_\varepsilon\omega^2\tau_0^2)\sin(2\omega t)], \tag{40}$$

where

$$a_1 = \kappa_\sigma + \kappa_\varepsilon + \frac{n+1}{2}\kappa_\sigma\kappa_\varepsilon, \quad a_2 = \frac{3\kappa_\varepsilon - \kappa_\sigma + 3d_1\kappa_\varepsilon^2 + d_2\kappa_\varepsilon^3}{(1+f_{3\sigma})(1+f_{1\sigma})}, \tag{41}$$

$$d_1 = \frac{1+n}{2} + \kappa_\sigma\frac{n(1-n)}{2},$$

$$d_2 = \frac{(1-n)^2}{4} + n^2 + \frac{n(1-n)}{2}\left(1 + \kappa_\sigma\frac{1+n}{2}\right), \tag{42}$$

$$b_0 = \frac{1}{2}\frac{(1+f_{3\varepsilon})(1+f_{1\varepsilon})}{(1+f_{3\sigma})(1+f_{1\sigma})},$$

$$b_1 = 2 + \kappa_\sigma\frac{1+n}{2}, \quad b_2 = 2 + \kappa_\varepsilon\frac{1+n}{2}, \tag{43}$$

$$L = (1+f_{3\sigma})(1+f_{1\sigma})(1+f_{3\varepsilon})(1+f_{1\varepsilon})(1+\omega^2\tau_1^2)(1+\omega^2\tau_3^2). \tag{44}$$

The Eqs. (39) and (40) yield the following expression for a total torque:

$$M = M_0\frac{3n-1}{2}[M_c + 2M_s\tan(\omega t)]\cos^2(\omega t), \tag{45}$$

where

$$M_c = \frac{\kappa_\varepsilon^2}{(1+f_{1\varepsilon})(1+f_{3\varepsilon})} + \frac{\kappa_\sigma - \kappa_\varepsilon}{L}(a_1 + a_2\omega^2\tau_0^2) \tag{46}$$

and

$$M_s = \frac{b_0\omega\tau_0}{L}(b_1\kappa_\sigma + b_2\kappa_\varepsilon\omega^2\tau_0^2)(\kappa_\sigma - \kappa_\varepsilon). \tag{47}$$

When $\omega\to 0$, $M_s \to 0$ and $M_c = \kappa_\sigma^2/(1+f_{1\sigma})(1+f_{3\sigma})$, i.e., expression (36) is recovered.

Equation (45) yields the following condition for the change of an orientation of a particle with respect to its orientation at the initial moment $t=0$:

$$M_c + 2M_s\tan(\omega t) < 0. \tag{48}$$

Using Eqs. (39) and (45) it can be shown that an inequality (48) is equivalent to the condition that $M(t)/M(0) < 0$. In the Appendix we demonstrated that this condition implies the change of the direction of the stable orientation of the ellipsoid.

Let us determine now a time interval inside the period of the external electric field T during which the condition (48) is satisfied. Consider only the case when $M_s<0$, e.g., $(\kappa_\sigma<\kappa_\varepsilon)$ and $M_c>0$. In this case the inequality (48) implies that there exist two time intervals where this inequality is satisfied:

$$\tan^{-1}\left(\frac{M_c}{2|M_s|}\right)<\omega t<\frac{\pi}{2}$$
$$\text{and } \pi+\tan^{-1}\left(\frac{M_c}{2|M_s|}\right)<\omega t<\frac{3\pi}{2}. \qquad (49)$$

During two time intervals determined by inequalities (49) the direction of the stable orientation becomes normal to the direction of the initial stable orientation, i.e., $\theta_s\to\theta_s+\pi/2$, where θ_s is an angle between the external electric field and the principal axis of the ellipsoid in the initial stable equilibrium. For a prolate spheroid $\theta_s=0$, while for an oblate spheroid $\theta_s=\pi/2$. Define two time intervals, T_1 and $T_2=T-T_1$. During time interval T_1 the stable orientation of the particle coincides with the initial stable orientation while during time interval T_2 [sum of two time intervals determined by Eqs. (49)] it changes. Equations (49) imply that

$$T_1=\frac{1}{\omega}\left(\pi+2\tan^{-1}\left(\frac{M_c}{2|M_s|}\right)\right),\quad T_2=\frac{2}{\omega}\tan^{-1}\left(\frac{2|M_s|}{M_c}\right). \qquad (50)$$

Assume that an ellipsoid is subjected to random perturbations uniformly distributed in time. Then a probability p_1 that an equilibrium orientation of the ellipsoid is the same as in the case of an ideal dielectric $p_1=T_1/(T_1+T_2)$ is

$$p_1=\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\left(\frac{M_c}{2|M_s|}\right), \qquad (51)$$

while the probability that its equilibrium orientation is normal to that in the case of an ideal dielectric $p_2=T_2/(T_1+T_2)$ is

$$p_2=\frac{1}{\pi}\tan^{-1}\left(\frac{2|M_s|}{M_c}\right). \qquad (52)$$

Consider a limiting case when $|M_s|\ll M_c$ and $\omega\tau_0\ll 1$. This situation occurs in the ranges $\kappa_\sigma\gg 1$, $\omega\tau_0\ll 1$ and $\kappa_\sigma\ll 1$, $\omega\tau_0\ll\kappa_\sigma$. In these limiting cases

$$M_s=\frac{b_0 b_1\kappa_\sigma(\kappa_\varepsilon-\kappa_\sigma)\omega\tau_0}{L},\quad M_c=\frac{\kappa_\sigma^2}{(1+f_{1\sigma})(1+f_{3\sigma})}, \qquad (53)$$

and

$$p_2=\frac{1}{\pi}\frac{\kappa_\varepsilon-\kappa_\sigma}{\kappa_\sigma}\frac{\omega\tau_0\left(2+\kappa_\sigma\frac{n+1}{2}\right)}{(1+f_{1\sigma})(1+f_{3\sigma})}. \qquad (54)$$

Thus the probability to detect an ellipsoid with an orientation normal to the orientation in the case of an ideal dielectric depends only upon the parameters of the system and the frequency of an applied electric field and does not depend upon its amplitude. It must be noted that independent of the magnitudes of M_c and $|M_s|$, $p_2<p_1$, $p_2<1/2$, and $p_1>1/2$.

V. CONCLUSIONS

We studied the moment of forces acting on a stationary dielectric ellipsoidal particle imbedded in a host dielectric medium with a finite electric conductivity under the action of a homogeneous, time independent or varying with time, electric field. Using a dipole moment approach we showed that in a constant electric field stable orientations of an ellipsoid for an ideal dielectric and a dielectric with a finite electric conductivity are the same.

We demonstrated that an equilibrium orientation of the ellipsoidal particle changes with time in a stationary electric field with a constant direction. It was found that during time interval T_1 an equilibrium orientation of the spheroidal particle with a finite electric conductivity remains the same as the equilibrium orientation of an ideal dielectric particle. During time interval T_2, where $T=T_1+T_2$ is a period of the external electric field, the equilibrium orientation of the axis of symmetry of the particle is normal to this direction.

The derived expressions for electric fields and currents [Eqs. (21) and (22)] and dipole moment [Eq. (31)] are valid also for the case of a rotating ellipsoid. In the latter case $x_i(0)$ and $E_{0i}(t)$ are spatial coordinates and components of electric field in a frame attached with the ellipsoid. Transformation into the laboratory frame is performed using the formulas $E_{0i}=O_{ik}(t)G_k(t)$ and $x_i=O_{im}\overline{x_m}$, where $G_k(t)$ and $\overline{x_m}$ are components of the electric field and coordinates in a laboratory frame. The orthogonal matrix O_{ik} is related with angular velocity $\vec{\Omega}$ by an equation $\dot{O}_{ik}=\varepsilon_{iml}\Omega_m O_{lk}$, where ε_{iml} is the Levi-Civita tensor. Using these formulas and expressions for $\Pi_i(t)$ and dipole moment $\vec{P}(t)$ allows to obtain equations governing the dynamics of a dipole moment similar to equations used in the studies of the Quincke rotation [13,14].

The obtained results imply a possibility of the existence of a mechanism of rotation of liquid particles that is alternative to the known effect of Quincke rotation. Indeed, the external electric field causes the deformation of a liquid particle along the direction of the field, and we showed that the direction of the equilibrium orientation of the ellipsoidal particle changes with time in a stationary external field.

In this study we considered an isotropic medium. The situation is completely different in the case of an anisotropic material. In the latter case the torque depends not only upon the orientation of the ellipsoid with respect to the direction of the external electric field but upon the mutual orientations of the principal axes of the medium, principal axes of the ellipsoid and the direction of the external electric field. Clearly, in this case our approach that uses the ellipsoidal coordinates and expansion of the external electric field into independent components directed along the principal axes of the ellipsoid is not valid. In this study we also considered only the case when $M_s<0$, e.g., $(\kappa_\sigma<\kappa_\varepsilon)$ and $M_c>0$. Analysis of this problem in other cases is a subject of ongoing investigation.

APPENDIX

Neglecting inertial effects the equation describing dynamics of an orientation of a spheroid in the vicinity of the equilibrium position reads

$$\eta\dot{\theta} = M(\theta), \tag{A1}$$

where a coefficient $\eta > 0$ depends upon the viscosity of a medium, M is a torque, and $\dot{\theta}$ is angular velocity.

Equation (A1) implies that the angle of a stable orientation θ_s is determined by two conditions:

$$M(\theta_s) = 0, \qquad \left.\frac{\partial M}{\partial \theta}\right|_{\theta=\theta_s} < 0. \tag{A2}$$

In the vicinity of the equilibrium

$$M(\theta) = \left.\frac{\partial M}{\partial \theta}\right|_{\theta=\theta_s} (\theta - \theta_s). \tag{A3}$$

In all cases considered in this study the dependence of the torque $M(\theta,t)$ can be written as

$$M(\theta,t) = M_0(\theta)\tilde{M}(t), \quad M_0 = \frac{\varepsilon_0\varepsilon_1 V E_0^2 \sin(2\theta)}{2}, \tag{A4}$$

where $\tilde{M}(t)$ is some normalization function that does not depend on angle θ [see Eqs. (34), (36), and (39)].

In the vicinity of the equilibrium position, where Eqs. (A2) and (A3) are valid, the change of the sign of $M(\theta,t)$ implies the change of the sign of the derivative $\partial M/\partial\theta$. Therefore if a particle had initially a stable orientation at $t=0$, then at time t_1 such that $M(\theta,t_1)/M(\theta,0) < 0$, the stable orientation of a particle is normal to its orientation at $t=0$. Therefore, Eq. (A4) implies that the change of the sign of a total torque $M(\theta,t)$ causes the change of the stable orientation to the direction normal to the initial one while the initially unstable orientation of the particle becomes stable.

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